6.3 Multiplication of a Vector by a Scalar

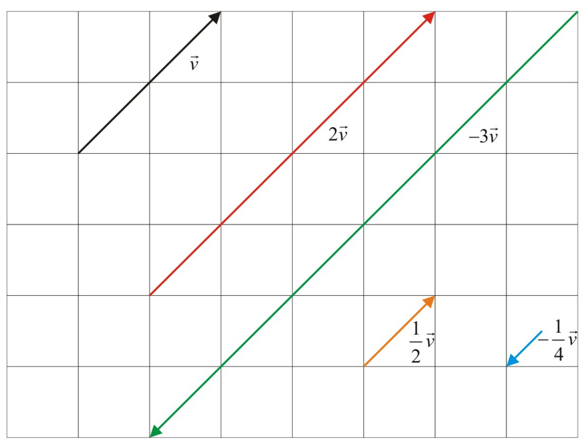
A Multiplication of a Vector by a Scalar

By multiplying a vector $\vec{v}$ by a scalar k we obtain a new vector noted $k\vec{v}$ with the following properties:

a) $k\vec{v}$ has the same direction as $\vec{v}$ if $k > 0$ and the opposite direction if $k < 0$

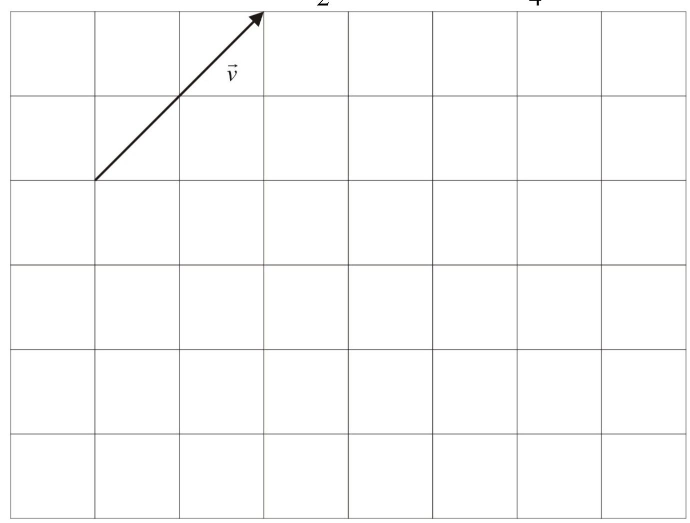
b) $\| k\vec{v} \| = |k| \times \| \vec{v} \|$

Examples:



Ex 1. Given the vector $\vec{v}$, draw the following vectors:

a) $2\vec{v}$ b) $-3\vec{v}$ d) $\frac{1}{2}\vec{v}$ e) $-\frac{1}{4}\vec{v}$



See the solution on the left side.

B Properties

The following properties apply for multiplication of a vector by a scalar:

$$k(\vec{a} + \vec{b}) = k\vec{a} + k\vec{b}$$
$$k(m\vec{a}) = (km)\vec{a} = km\vec{a}$$
$$(k + m)\vec{a} = k\vec{a} + m\vec{a}$$

Ex 2. Given $\vec{a} = 2\vec{i} - 3\vec{j} + \vec{k}$, $\vec{b} = -\vec{i} + \vec{j} + 2\vec{k}$, write the following expressions in terms of the vectors $\vec{i}$, $\vec{j}$, and $\vec{k}$.

a) $\vec{a} + \vec{b}$

$$\vec{a} + \vec{b} = (2\vec{i} - 3\vec{j} + \vec{k}) + (-\vec{i} + \vec{j} + 2\vec{k})$$
$$= (2-1)\vec{i} + (-3+1)\vec{j} + (1+2)\vec{k} = \vec{i} - 2\vec{j} + 3\vec{k}$$

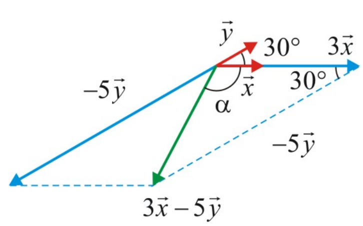
b) $2\vec{a} - 3\vec{b}$

$$2\vec{a} - 3\vec{b} = 2(2\vec{i} - 3\vec{j} + \vec{k}) - 3(-\vec{i} + \vec{j} + 2\vec{k})$$
$$= 4\vec{i} - 6\vec{j} + 2\vec{k} + 3\vec{i} - 3\vec{j} - 6\vec{k}$$
$$= (4+3)\vec{i} + (-6-3)\vec{j} + (2-6)\vec{k} = 7\vec{i} - 9\vec{j} - 4\vec{k}$$

C Vector Unit

An unit vector is a vector having a magnitude of 1. For any vector $\vec{v}$, a unit vector parallel to $\vec{v}$ is given by:

$$\vec{u} = \frac{\vec{v}}{\| \vec{v} \|}$$

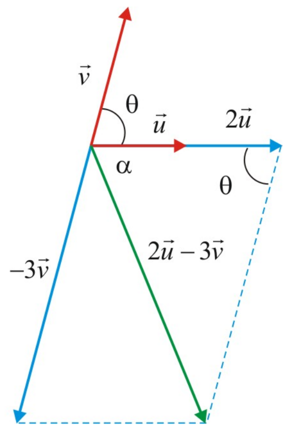


Ex 3. If $\vec{x}$ and $\vec{y}$ are two unit vectors with an angle of $30°$ between them, find the magnitude and direction of the vector $3\vec{x} - 5\vec{y}$.

$$\| 3\vec{x} - 5\vec{y} \|^2 = \| 3\vec{x} \|^2 + \| 5\vec{y} \|^2 - 2 \| 3\vec{x} \| \| 5\vec{y} \| \cos 30°$$
$$\| 3\vec{x} \| = 3 \| \vec{x} \| = 3$$
$$\| 5\vec{y} \| = 5 \| \vec{y} \| = 5$$
$$\| 3\vec{x} - 5\vec{y} \|^2 = 3^2 + 5^2 - 2(3)(5)\cos 30° = 34 - 15\sqrt{3}$$
$$\| 3\vec{x} - 5\vec{y} \| = \sqrt{34 - 15\sqrt{3}} \cong 2.83$$

According to the left diagram:

$$\frac{\| -5\vec{y} \|}{\sin \alpha} = \frac{\| 3\vec{x} - 5\vec{y} \|}{\sin 30°} \Rightarrow \alpha = \sin^{-1}\left(\frac{5 \sin 30°}{\sqrt{34 - 15\sqrt{3}}} \right) = 61.98°$$



Ex 4. Given $\|\vec{u}\| = 8m$ and $\|\vec{v}\| = 12m$, $\|\vec{u}+\vec{v}\| = 16$, determine the magnitude and the direction of the vector $2\vec{u}-3\vec{v}$.

$$\theta = \angle(\vec{u},\vec{v}) = \angle(2\vec{u},3\vec{v})$$

$$\|\vec{u}+\vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2 + 2\|\vec{u}\|\|\vec{v}\|\cos\theta$$

$$\cos\theta = \frac{\|\vec{u}+\vec{v}\|^2 - \|\vec{u}\|^2 - \|\vec{v}\|^2}{2\|\vec{u}\|\|\vec{v}\|} = \frac{16^2 - 8^2 - 12^2}{2(8)(12)} = \frac{1}{4}$$

$$\|2\vec{u}-3\vec{v}\|^2 = \|2\vec{u}\|^2 + \|3\vec{v}\|^2 - 2\|2\vec{u}\|\|3\vec{v}\|\cos\theta$$

$$= 4\|\vec{u}\|^2 + 9\|\vec{v}\|^2 - 2(2)(3)\|\vec{u}\|\|\vec{v}\|\cos\theta$$

$$= 4(8^2) + 9(12^2) - 12(8)(12)(1/4) = 1264$$

$$\|2\vec{u}-3\vec{v}\| = \sqrt{1264} = 35.55$$

According to the diagram on the left side:

$$\frac{\|-3\vec{v}\|}{\sin\alpha} = \frac{\|2\vec{u}-3\vec{v}\|}{\sin\theta} \Rightarrow \alpha = \sin^{-1}\left(\frac{3(12)\sqrt{1-1/16}}{\sqrt{1264}}\right) = 78.64°$$

Reading: Nelson Textbook, Pages 293-298
Homework: Nelson Textbook: Page 298 #4, 9, 13, 15, 17, 18, 21, 22

